"WHY THE PROFESSOR CAN'T TEACH"

D'aquest llibre de Morris Klein n'hem estret dos capítols particularment interessants, que reproduim, respectivament, en aquest i en el proper butlletí.

The Vicious Circle

In a self-centered circle, he goes round and round,
That he is a wonder is true;
For who but an egotist ever could be
Circumference and center, too.

Sarah Fells

Peter Landers found himself caught in a vicious circle. He had just secured a Ph.D. in mathematics from Prestidigious University and, having been well recommended, readily secured a faculty position at Admirable University. Thereupon Peter faced the problem of teaching mathematics to prospective engineers, social scientists, physicists, elementary and secondary school teachers, the general liberal arts students, and those who, like himself, had chosen to become mathematicians. Peter was fully aware of these varied career interests, and he also knew that students came to college with different drives and preparation. But he was confident that his education, typical for Ph.D.'s, had prepared him for the tasks ahead.

To put himself in the proper frame of mind he reviewed his own education. The elementary school courses had been acceptable. After all, one did have to know how much to pay

for five candy bars if he knew the price of a single bar. True, some operations were baffling. It had not been clear why the division of two fractions had to be performed by inverting the denominator and multiplying—but the teacher seemed to know what was correct. He had constantly referred to rules, principles, and laws. Rules, like rules of behavior, apparently applied to arithmetic, too. For all Peter had known, principles were laid down by the principals of the schools, and certainly *they* were authorities. As for laws, everyone knew that there were city laws, state laws, federal laws, and even the laws of the Ten Commandments. Certainly laws must be obeyed. Though under some tension as to whether he was violating laws, Peter was young and resilient. In any case, what to do was clear and the answers were right.

In his review of his high school education Peter did recall some doubts he had had about the value of what he was being taught. He hadn't understood why the teacher had to stress that the sum of two whole numbers is a whole number, or why he had to prove that there is one and only one midpoint on every line segment; but evidently the teacher was trying to make sure that no one could be mistaken on these elementary matters. After all, teachers knew best what had to be done.

Peter also recalled one teacher's enthusiasm about the quadratic formula. "You see," the teacher proclaimed triumphantly after he had derived the formula, "we can now solve any quadratic equation." But Peter had been perverse and had asked the teacher why anyone wanted to solve any quadratic equation. The teacher's reply was a disdainful look that caused Peter to shrink back. His question must have been a silly one.

He remembered a similar experience in geometry. After a long and apparently strenuous effort, the teacher proved

that two triangles are congruent if the sides of one are equal respectively to the sides of the other. Then he turned to the class as if expecting applause. Again Peter dared to speak up: "But isn't that obvious? A triangle is a rigid figure. If you put three sticks together to form a triangle, you cannot change its size or shape." Peter had learned this at the age of five while playing with Erector sets. The teacher's contempt was obvious. "Who's talking about sticks? We are concerned with triangles."

Despite a few other disagreeable incidents Peter continued to like mathematics. He believed in his teachers. It was easy to comply with their requests, and the certitude of the results gave him, as they had given others before him, immense satisfaction. And so Peter moved on to college with the conviction that he liked mathematics and was going to major in it.

His first experiences were disturbing. After his program was approved by an adviser who did not understand what an Advanced Placement Examination Grade of 4.5 meant—the adviser had thought that 10 was a perfect grade so that 4.5 was a poor one—Peter was finally registered.

He entered his first college classroom for a course which happened to be English. To his surprise he found about five hundred students already seated. The professor arrived, delivered his lecture, and, obviously very busy, rushed out of the room. Peter never found out what his name was, but apparently names were not important, because the professor never bothered to ask any student his name either. Nor, Peter thought, would the professor have noticed had a different group of five hundred students appeared each time. Term papers were required, and these were graded by graduate students who insisted that "Who shall I call next?" was correct, though Peter had been taught otherwise in high school. The size of the class and the impersonal character of

the instruction disturbed Peter at first, but he soon realized that the requirements of the English course could be met merely by listening. And so he relaxed.

Peter's second class, one in social science, surprised him for different reasons. At the professor's desk was a young man not much older than Peter. As the instructor conducted the lesson he was obviously nervous. Somehow the lessons throughout the semester were confined almost entirely to the first part of the text. And the instructor did not welcome questions.

The third class—mathematics—was a shock. Peter entered the room and found that it was a large auditorium. At the bottom of the room, at the professor's desk, was not a man but a box, which proved to be a television set. Shortly after Peter's entrance the box began to speak and the students took notes feverishly. From many seats one could not see clearly, if at all. But by coming early one could get a good seat. And so Peter managed to learn some of his college mathematics by listening and looking at a TV program.

Though it was not a requirement, Peter decided to take some physics. He had heard somewhere that mathematics was applied to physics, and he thought he should find out what these applications were. The physics professor constantly talked about infinitesimals and which infinitesimals could be neglected. The mathematics professors, however, had warned that such concepts and procedures were loose and even incorrect. But Peter listened attentively. He was sure that even though the mathematics and the physics professors apparently did not communicate with each other and so did not talk the same language, their methodologies could be reconciled. He did seek counsel from his professors on this matter, but unfortunately they were not available. One was actually living out of the city, in Washington, D.C.; another was always involved in consulta-

tions outside the university; and a third had office hours only on Sundays, from 6:00 to 8:00 A.M.

In the junior and senior years the classes were smaller, and the courses were usually taught by older faculty. Many blithely ignored the texts they had assigned and spent the period transferring material from their notes to the board. The professors copied assiduously and the students did likewise. When the professors looked up from their notes they looked into the blackboard as though the students were behind it.

Nevertheless Peter persevered, received his bachelor's degree, and proceeded to graduate school. His experiences there paralleled those of most other students. Professors were hard to contact. The bulletin descriptions of the courses bore no relation to what the professors taught. Each professor presented his own specialty as though nothing had been done or was being done by anyone else in the world. And so Peter learned about categories, infinite Abelian groups, diffeomorphisms, noncommutative rings, and a variety of other specialties.

Prospective Ph.D.'s must write a doctoral thesis. Finding a thesis adviser was like hunting for water in a desert. After many trials, including writing theses on topics suggested by his professor that, it turned out, had been done elsewhere and even published, Peter wrote a thesis on almost perfect numbers that completed his work for the degree.

With the Ph.D. behind him, Peter presumed he was prepared for college teaching. Upon taking up his position at Admirable University he received from his department chairman the syllabi for the several courses he was to teach and was told what texts he was to use for these courses. Cheerful, personable Peter went about his assigned tasks with enthusiasm. He had always liked mathematics and had no doubt that he could convey his enthusiasm and

understanding of the subject to his students. He had been informed by the chairman that to secure promotion and tenure he would be expected to do research. This requirement in no way dimmed Peter's spirit, because he had been told repeatedly that mathematicians do research and was confident that the training he had received had prepared him for it.

But the world soon began to close in on Peter. As a novice he was assigned to teach freshmen and sophomores. His first course was for liberal arts students, that is, students who do not intend to use mathematics professionally but who take it either to meet a requirement for a degree or just to learn more about the subject. Recognizing that many of these students are weak in algebra, Peter thought he would review negative numbers. To make these numbers meaningful he reminded the students that they are used to represent temperatures below zero; and to emphasize the physical significance of negative temperatures he pointed out that water freezes at 32° F., so that a negative temperature means a state far below freezing. Though the example was pedagogically wise, Peter could see at once that the students' minds had also frozen, and the rest of his lesson could not penetrate the ice.

In a later lesson Peter tried another subject. As an algebraist by preference he thought students would enjoy learning about a novel algebra. There is an arithmetic that reduces all whole numbers by the nearest multiple of twelve. To make his lesson concrete Peter presented clock arithmetic as a practical example: Clocks ignore multiples of twelve, so that four hours after ten o'clock is two o'clock. The mere mention of clocks caused the students to look at their watches, and it was obvious that they were counting the minutes until the end of the period.

And so Peter tried another novelty, the Koenigsberg

bridge problem. Some two hundred years ago the citizens of the village of Koenigsberg in East Prussia became intrigued with the problem of crossing seven nearby bridges in succession without recrossing any. The problem attracted Leonhard Euler, the eighteenth century's greatest mathematician, and he soon showed by an ingenious trick that such a path was impossible. The villagers, who did not know this, continued for years to amuse themselves by making one trip after another during their walks on sunny afternoons—but when Peter presented the problem in the artificial, gloomy light of the classroom, a chill descended on the class.

Peter's next class was a group of pre-engineering students. These students, he was sure, would appreciate mathematics, and so he introduced the subject of Boolean algebra. This algebra, created by the mathematician and logician George Boole, does have application to the design of electric circuits. The mention of electric circuits appeared to arouse some interest, and so Peter explained Boolean algebra. But then one student asked Peter how one uses the algebra to design circuits. Unfortunately, Peter's training had been in pure mathematics and he did not know how to answer the question. He was compelled to admit this and detected obvious signs of disappointment and hostility in the students. They evidently believed that they had been tricked. In his attempts to explain and clarify other mathematical themes Peter also learned that engineering students cared only about rules they could use for building things. Mathematics proper was of no interest.

Nor were the premedical students any more kindly disposed to mathematics. Their attitude was that doctors do not use mathematics but take it only because it is required for the physics course, and even the physics seemed of dubious value. The physical and social scientists had a similar attitude. Mathematics was a tool. They were interested in the

real world and in real people, and certainly mathematics was not part of that reality.

Peter was soon called upon to teach prospective elementary and high school teachers. He did not expect much of the former. These students were preparing to teach many different subjects and so could not take a strong interest in mathematics. However, high school teachers specialize in one area, and Peter certainly expected them to appreciate what he had to offer. But every time he introduced a new topic, the first question the students asked was, "Will we have to teach this?" Peter did not know what the high schools were currently teaching or what they were likely to teach in any changes impending in the high school curriculum. Hence, he honestly answered either "No" or "I don't know." Upon hearing either response the prospective teachers withdrew into their shells, and Peter's teachings were reflected from impenetrable surfaces.

Peter's one hope for a response to his enthusiasm for teaching was the mathematics majors. Surely they would appreciate what he had to offer. But even these students seemed to want to "get it over with." If he presented a theorem and proof, they noted them carefully and could repeat them on tests; however, any discussion of why the theorem was useful or why one method of proof was likely to be more successful or more desirable than another bored them.

A couple of years of desperate but fruitless efforts caused Peter to sit back and think. He had projected himself and his own values and he had failed. He was not reaching his students. The liberal arts students saw no value in mathematics. The mathematics majors pursued mathematics because, like Peter, they were pleased to get correct answers to problems. But there was no genuine interest in the subject. Those students who would use mathematics in some

profession or career insisted on being shown immediately how the material could be useful to them. A mere assurance that they would need it did not suffice. And so Peter began to wonder whether the subject matter prescribed in the syllabi was really suitable. Perhaps, unintentionally, he was wasting his students' time.

Peter decided to investigate the value of the material he had been asked to teach. His first recourse was to check with his colleagues, who had taught from five to twenty-five or more years. But they knew no more than Peter about what physical scientists, social scientists, engineers, and high school and elementary school teachers really ought to learn. Like himself, they merely followed syllabi—and no one knew who had written the syllabi.

Peter's next recourse was to examine the textbooks in the field. Surely professors in other institutions had overcome the problems he faced. His first glance through publishers' catalogues cheered him. He saw titles such as *Mathematics for Liberal Arts, Mathematics for Biologists, Calculus for Social Scientists*, and *Applied Mathematics for Engineers.* He eagerly secured copies. But the texts proved to be a crushing disappointment. Only the authors' and publishers' names seemed to differentiate them. The contents were about the same, whether the authors in their prefaces or the publishers in their advertising literature professed to address liberal arts students, prospective engineers, students of business, or prospective teachers. Motivation and use of the mathematics were entirely ignored. It was evident that these authors had no idea of what anyone did with mathematics.

Clearly a variety of new courses had to be fashioned and texts written that would present material appropriate for the respective audiences. The task was, of course, enormous, and it was certain that it could not be accomplished by one man over a few years' time. Nevertheless Peter became

enthusiastic about the prospect of interesting investigations and writing that would lure students into the study of mathematics and endear it to them. The spirit of the teacher arose and swelled within him. As these pleasant thoughts swirled through his mind, another, dampening thought, like a dark cloud on the horizon, soon entered. He was a recently appointed professor. Promotion and, more important, tenure were yet to be secured. Without these his efforts to improve teaching would be pointless—he would be unable to put the product of his work to use. But promotion and tenure were obtained through research in some highly advanced and recondite problems almost necessarily chosen in the only field in which he had acquired some competence through his doctoral work. Such research was no minor undertaking. It demanded full time and total effort.

Clearly, he must give the research precedence, and then perhaps he could undertake the improvement of teaching. And so for practical reasons Peter decided to devote the next few years to research. But the struggle to publish and to remain in the swim for promotion and salary increases caught Peter in a vortex of never-ending spirals of motion; and the closer he came to the center the deeper he was sucked into research. In the meantime Peter continued to teach in accordance with the syllabi and texts handed down to him by his chairman. His few, necessarily limited efforts to stir up some activity among his older colleagues, who were in a better position to break from the existing patterns, were futile because these professors had accepted the existing state of affairs and chose to shine in research. Success there was more prestigious and more lucrative.

Ultimately, Peter, like other human beings, succumbed to the lures that prominence in research held forth. As for the students—well, students came and went, and they soon

became vague faces and unremembered names. Education might hope for an epiphany, but Peter was not ordained to be the god of educational reformation. By the time he had acquired tenure he had joined the club. Like others before him he concentrated on research and the training of future researchers who would also be compelled to resort to perfunctory and ineffective teaching. Peter had taken his place in the vicious circle.

The history of Peter Landers' aborted teaching efforts, real enough, seems exaggerated. One might conceive of its taking place in nineteenth-century Germany or France. But the United States is devoted to education. We were the first nation to espouse universal education and to foster the realization of the potential of every youth. Our Founding Fathers, notably Benjamin Franklin and Thomas Jefferson, stressed the necessity of this policy, and it was adopted. Even today no country matches the educational opportunities and facilities that the United States provides for its youth. But the practices within educational institutions seem to be in marked variance with the principles and policies of our country.

How has it come to pass that Peter and the many thousands of his colleagues find themselves enslaved by research, while education, the major goal of our vast educational system, is being sacrificed? Does the pressure to do research stem from the professors because they prefer the prestige and monetary rewards? Or does it come from the university administrations? In either case, does not research make for better teaching? Or is there a conflict between the two, and if there is, how can we resolve it? Since the crux of the problem lies with the universities—which train the teachers of all educational disciplines and at all levels—we must examine the policies and practices of our higher educational institutions.