### Publicacions més rellevants de la línia de recerca:
## Objectes invariants en sistemes dinàmics i llurs connexions

**Referència:** Delshams, A. Llave, R. de la and Seara, T.M. Geometric properties of the scattering map of a normally hyperbolic invariant manifold. *Adv. Math.*, **217(3)** (2008), pp. 1096-1153.

**Abstract:** Given a normally hyperbolic invariant manifold $\Lambda$ for a map $f$, whose stable and unstable invariant manifolds intersect transversally, we consider its associated scattering map. That is, the map that, given an asymptotic orbit in the past, gives the asymptotic orbit in the future. We show that when $f$ and $\Lambda$ are symplectic (respectively exact symplectic) then, the scattering map is symplectic (respectively exact symplectic). Furthermore, we show that, in the exact symplectic case, there are extremely easy formulas for the primitive function, which have a variational interpretation as difference of actions. We use this geometric information to obtain efficient perturbative calculations of the scattering map using deformation theory. This perturbation theory generalizes and extends several results already obtained using the Melnikov method. Analogous results are true for Hamiltonian flows. The proofs are obtained by geometrically natural methods and do not involve the use of particular coordinate systems, hence the results can be used to obtain intersection properties of objects of any type. We also reexamine the calculation of the scattering map in a geodesic flow perturbed by a quasi-periodic potential. We show that the geometric theory reproduces the results obtained in [Amadeu Delshams, Rafael de la Llave, Tere M. Seara, Orbits of unbounded energy in quasi-periodic perturbations of geodesic flows, Adv. Math. 202 (1) (2006) 64188] using methods of fastslow systems. Moreover, the geometric theory allows to compute perturbatively the dependence on the slow variables, which does not seem to be accessible to the previous methods.

---

**Referència:**

Morales-Ruiz, J.J. and Simon, S. On the meromorphic non-integrability of some $N$-body problems. *Discrete Contin. Dyn. Syst.* **24(4)** (2009), pp. 1225–1273.

**Abstract:** We present a proof of the meromorphic non–integrability of the planar $N$-Body Problem for some special cases. A simpler proof is added to those already existing for the Three-Body Problem with arbitrary masses. The $N$-Body Problem with equal masses is also proven non-integrable. Furthermore, a new general result on additional integrals is obtained which, applied to these specific cases, proves the non-existence of an additional integral for the general Three-Body Problem, and provides for an upper bound on the amount of additional integrals for the equal-mass

Problem for $N = 4, 5, 6$. These results appear to qualify differential Galois theory, and especially a new incipient theory stemming from it, as an amenable setting for the detection of obstructions to Hamiltonian integrability.

---

**Referència:**

Ollé, M., Pacha, J.R., and Villanueva, J. Kolmogorov-Arnold-Moser aspects of the periodic Hamiltonian Hopf bifurcation. *Nonlinearity* **21(8)** (2008), pp. 1759–1811.

**Abstract:**

In this work we consider a $1 : -1$ non-semi-simple resonant periodic orbit of a three degrees of freedom real analytic Hamiltonian system. From the formal analysis of the normal form, we prove the branching off of a two-parameter family of two-dimensional invariant tori of the normalized system, whose normal behaviour depends intrinsically on the coefficients of its low-order terms. Thus, only elliptic or elliptic together with parabolic and hyperbolic tori may detach from the resonant periodic orbit. Both patterns are mentioned in the literature as the direct and inverse, respectively, periodic Hopf bifurcation. In this paper we focus on the direct case, which has many applications in several fields of science. Our target is to prove, in the framework of Kolmogorov. Arnold.Moser (KAM) theory, the persistence of most of the (normally) elliptic tori of the normal form, when the whole Hamiltonian is taken into account, and to give a very precise characterization of the parameters labelling them, which can be selected with a very clear dynamical meaning. Furthermore, we give sharp quantitative estimates on the "density" of surviving tori, when the distance to the resonant periodic orbit goes to zero, and show that the four-dimensional invariant Cantor manifold holding them admits a Whitney-$C^\infty$ extension. Due to the strong degeneracy of the problem, some standard KAM methods for elliptic low-dimensional tori of Hamiltonian systems do not apply directly, so one needs to properly suit these techniques to the context.

---

**Referència:** Lomelí, H., Meiss, J.D. and Ramírez-Ros, R. Canonical Melnikov theory for diffeomorphisms. *Nonlinearity* **21(3)** (2008), pp 485–508.

**Abstract:** We study perturbations of diffeomorphisms that have a saddle connection between a pair of normally hyperbolic invariant manifolds. We develop a first-order deformation calculus for invariant manifolds and show that a generalized Melnikov function or Melnikov displacement can be written in a canonical way. This function is defined to be a section of the normal bundle of the saddle connection.

We show how our definition reproduces the classical methods of Poincaré and Melnikov and specializes to methods previously used for exact symplectic and volume-preserving maps. We use the method to detect the transverse intersection of stable and unstable manifolds and relate this intersection to the set of zeros of the Melnikov displacement.