Publicacions més rellevants de la línia de recerca:

Equacions de reacció difusió

Referència: Cabré, X. and Solà-Morales, J. Layer solutions in a half-space for boundary reactions, *Comm. Pure Appl. Math.*, **58** (2005), pp. 1678–1732.

Abstract: We consider harmonic functions in a halfspace $\mathbb{R}^n_+ = \{(x,y) \in \mathbb{R}\times\mathbb{R}^{n-1} : x > 0\}$ subject to nonlinear Neumann boundary conditions. We study bounded solutions which are monotone increasing from -1 to 1 in one of the y-variables. We call such functions *layer solutions*. When $n = 2$, we establish that a necessary and sufficient condition for the existence of a layer solution is that the boundary energy potential (the primitive of the nonlinearity, up to a sign) has two, and only two, absolute minima in the interval $[-1, 1]$, located at ± 1. In addition, we prove uniqueness of the layer solution up to translations in the y-variable. When $n = 3$, we establish that every stable solution in $\mathbb{R}^3_+$ (and in particular, every layer solution, and every local minimizer) is in fact a function of only two variables.

Referència: Cabré, X. and Capella, A. Regularity of radial minimizers and extremal solutions of semilinear elliptic equations, *Journal of Functional Analysis*, **238** (2006), pp. 709–733.

Abstract: We consider a special class of radial solutions of semilinear equations $-\Delta u = g(u)$ in the unit ball of $\mathbb{R}^n$. It is the class of semi-stable solutions, which includes local minimizers, minimal solutions, and extremal solutions. We establish sharp pointwise, L^q, and $W^{k,q}$ estimates for semi-stable radial solutions. Our regularity results do not depend on the specific nonlinearity g.

Among other results, we prove that every semi-stable radial weak solution $u \in H^1_0$ is bounded if $n \leq 9$ (for every g), and belongs to $H^3 = W^{3,2}$ in all dimensions n (for every g increasing and convex). The optimal regularity results are strongly related to an explicit exponent which is larger than the critical Sobolev exponent.

Referència: Cabré, X. and Terra, J. Saddle-shaped solutions of bistable diffusion equations in all of $\mathbb{R}^{2m}$, *J. European Math. Society*, **11** (2009), pp. 819–843.

Abstract: We study the existence and instability properties of saddle-shaped solutions of the semilinear elliptic equation $-\Delta u = f(u)$ in the whole $\mathbb{R}^{2m}$, where f is of bistable type. It is known that in dimension $2m = 2$ there exists a saddle-shaped solution. This is a solution which changes sign in $\mathbb{R}^2$ and vanishes only on $\{|x_1| = |x_2|\}$. It is also known that this solution is unstable.

In this article we prove the existence of saddle-shaped solutions in every even dimension, as well as their instability in the case of dimension $2m = 4$. More precisely, our main result establishes that if $2m = 4$, every solution vanishing on the Simons cone $\{(x^1, x^2) \in \mathbb{R}^m \times \mathbb{R}^m : |x^1| = |x^2|\}$ is unstable outside of every compact set and, as a consequence, has infinite Morse index. These results are relevant in connection with a conjecture of De Giorgi extensively studied in recent years and for which the existence of a counter-example in high dimensions is still an open problem.