## Publicacions més rellevants de la línia de recerca: Tècniques variacionals en problemes de contorn

**Referència:** Bendito, E., Carmona, Á., Encinas, A.M. and Gesto, J.M. Estimation of Fekete points. *Journal of Computational Physics*, **225** (2007), pp. 2354–2376.

Abstract: We aim here at presenting a new procedure to numerically estimate the Fekete points of a wide variety of compact sets in  $\mathbb{R}^3$ . We understand the Fekete point problem in terms of the identification of near equilibrium configurations for a potential energy that depends on the relative position of N particles. The compact sets for which our procedure works are basically the finite union of piecewise regular surfaces and curves. In order to determine a good initial configuration to start the search of the Fekete points of these objects, we construct a sequence of approximating regular surfaces. Our algorithm is based on the concept of disequilibrium degree, which is defined from a physical interpretation of the behavior of a system of particles when they search for a minimum energy configuration. Moreover, the algorithm is efficient and robust independently of the considered compact set as well as of the kernel used to define the energy. The numerical experimentation carried out suggests that a local minimum can be localized with a computational cost of order less than  $N^3$ .

**Referència:** Bendito, E., Carmona, Á. and Encinas, A.M. Boundary Value Problems on Weighted Networks. *Discrete Applied Mathematics*, **156** (2008), pp. 3443–3463.

**Abstract:** We present here a systematic study of general boundary value problems on weighted networks that includes the variational formulation of such problems. In particular, we obtain the discrete version of the Dirichlet Principle and we apply it to the analysis of the inverse problem of identifying the conductivities of the network in a very general framework. Our approach is based on the development of an efficient vector calculus on weighted networks which mimetizes the calculus in the smooth case. The key tool is an adequate construction of the tangent space at each vertex. This allows us to consider discrete vector fields, inner products and general metrics. Then, we obtain discrete versions of derivative, gradient, divergence and Laplace–Beltrami operators, satisfying analogous properties to those verified by their continuous counterparts. On the other hand we develop the corresponding integral calculus that includes the discrete vector calculus to analyze the consistency of difference schemes used to solve numerically a Robin boundary value problem in a square.

**Referència:** Bendito, E., Carmona, Á., Encinas, A.M. and Gesto, J.M. Characterization of symmetric *M*-matrices as resistive inverses. *Linear Algebra and its Applications*, **430** (2009), pp. 1336–1349.

**Abstract:** We aim here at characterizing those non-negative matrices whose inverse is an irreducible Stieltjes matrix. Specifically, we prove that any irreducible Stieltjes matrix is a resistive inverse. To do this we consider the network defined by the off-diagonal entries of the matrix and we identify the matrix with a positive definite Schrödinger operator whose ground state is determined by the lowest eigenvalue of the matrix and the corresponding positive eigenvector. We also analyze the case in which the operator is positive semi-definite which corresponds to the study of singular irreducible symmetric M-matrices.