

**Publicacions més rellevants de la línia de recerca:**  
**Problemes extremals en Combinatòria i Teoria de Grafs**

**Referència:** Balbuena, C. and García-Vázquez, P. On the minimum order of extremal graphs to have a prescribed girth. *SIAM J. Discrete Math.*, **21(1)** (2007), pp. 251–257.

**Abstract:** We show that any  $n$ -vertex extremal graph  $G$  without cycles of length at most  $k$  has girth exactly  $k + 1$  if  $k \geq 6$  and  $n > (2(k - 2)^{k-2} + k - 5)/(k - 3)$ . This result provides an improvement of the known asymptotic result by F. Lazebnik and P. Wang [*J. Graph Theory* 26 (1997), no. 3, 147-153], who proved that the girth is exactly  $k + 1$  if  $k \geq 12$  and  $n \geq 2^{a^2+a+1}k^a$ , where  $a = k - 3 - \lfloor (k - 2)/4 \rfloor$ . Moreover, we prove that the girth of  $G$  is at most  $k + 2$  if  $n > (2(t - 2)^{k-2} + t - 5)/(t - 3)$ , where  $t = \lceil (k + 1)/2 \rceil \geq 4$ . In general, for  $k \geq 5$  we show that the girth of  $G$  is at most  $2k - 4$  if  $n \geq 2k - 2$ .

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**Referència:** Balbuena, C. Incidence matrices of projective planes and of some regular bipartite graphs of girth 6 with few vertices. *SIAM J. Discrete Math.*, **22(4)** (2008), pp. 1351–1363.

**Abstract:** In this paper, two Latin squares with entries from  $\{0, 1, \dots, n\}$  are defined to be quasi row-disjoint if and only if the cartesian product of any two rows contains at most one pair  $(x, x)$  with  $x \neq 0$ . The main result of this work is a method for constructing a family of  $q$  mutually quasi row-disjoint Latin squares. From this family we obtain in a very easy way the incidence matrices of projective planes and affine planes of order  $q$  where  $q$  is a prime power. Also the incidence matrices of  $(q - r)$ -regular balanced bipartite graphs of girth 6 and order  $2(q^2 - rq - 1)$  for  $r = 0, 1, \dots, q - 3$  are obtained. Moreover, this result for  $r = 1$  is improved, finding  $(q - 1)$ -regular bipartite graphs of girth 6 with  $q^2 - q - 2$  vertices in each partite set. Some of these graphs have the smallest number of vertices known so far among the regular graphs with girth 6.

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**Referència:** Tang, J., Lin, Y., Balbuena, C. and Miller, M. Calculating the extremal number  $ex(v; C_3, C_4, \dots, C_n)$ . *Discrete Appl. Math.*, **157(9)** (2009), pp. 2198–2206.

**Abstract:** By the extremal number  $ex(v; \{C_3, C_4, \dots, C_n\})$  we denote the maximum number of edges in a graph of order  $v$  and girth at least  $g \geq n + 1$ . The set of such graphs is denoted by

$EX(v; \{C_3, C_4, \dots, C_n\})$ . In 1975, Erdős mentioned the problem of determining extremal numbers  $ex(v; \{C_3, C_4\})$  in a graph of order  $v$  and girth at least 5. In this paper, we consider a generalized version of the problem for any value of girth by using the hybrid simulated annealing and genetic algorithm (HSAGA). Using this algorithm, some new results for  $n \geq 5$  have been obtained. In particular, we generate some graphs of girth 6, 7 and 8 which in some cases have more edges than corresponding cages. Furthermore, future work will be described regarding the investigation of structural properties of such extremal graphs and the implementation of HSAGA using parallel computing.