

## Publicacions més rellevants de la línia de recerca: Desigualtats amb pesos

**Referència:** Carro, M.J., Gogatishvili, A., Martín, J., Pick, L. Functional properties of rearrangement invariant spaces defined in terms of oscillations. *J. Funct. Anal.*, **229(2)** (2005), pp. 375–404.

**Abstract:** Function spaces whose definition involves the quantity  $f^{**} - f^*$ , which measures the oscillation of  $f^*$ , have recently attracted plenty of interest and proved to have many applications in various, quite diverse fields. Primary role is played by the spaces  $S_p(\omega)$ , with  $0 < p < \infty$  and  $\omega$  a weight function on  $(0, \infty)$ , defined as the set of Lebesgue-measurable functions on  $\mathbb{R}$  source such that  $f^*(\infty) = 0$  and

$$\|f\|_{S_p(\omega)} := \left( \int_0^\infty (f^{**}(s) - f^*(s))^p \omega(s) \, ds \right)^{1/p} < \infty$$

Some of the main open questions concerning these spaces relate to their functional properties, such as their lattice property, normability and linearity. We study these properties in this paper.

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**Referència:** Carro, M.J., Raposo, J.A., Soria, J. Recent developments in the theory of Lorentz spaces and weighted inequalities. *Mem. Amer. Math. Soc.*, **187** (2007).

**Abstract:** The main objective of this work is to bring together two well known and, a priori, unrelated theories dealing with weighted inequalities for the Hardy-Littlewood maximal operator  $M$ . For this, the authors consider the boundedness of  $M$  in the weighted Lorentz space  $\Lambda_u^p(w)$ . Two examples are historically relevant as a motivation: If  $w = 1$ , this corresponds to the study of the boundedness of  $M$  on  $L^p(u)$ , which was characterized by B. Muckenhoupt in 1972, and the solution is given by the so called  $A_p$  weights. The second case is when we take  $u = 1$ . This is a more recent theory, and was completely solved by M.A. Ariño and B. Muckenhoupt in 1991. It turns out that the boundedness of  $M$  on  $\Lambda^p(w)$  can be seen to be equivalent to the boundedness of the Hardy operator  $A$  restricted to decreasing functions of  $L^p(w)$ , since the nonincreasing rearrangement of  $Mf$  is pointwise equivalent to  $Af^*$ . The class of weights satisfying this boundedness is known as  $B_p$ . Even though the  $A_p$  and  $B_p$  classes enjoy some similar features, they come from very different theories, and so are the techniques used on each case: Calderon-Zygmund decompositions and covering lemmas for  $A_p$ , rearrangement invariant properties and positive integral operators for  $B_p$ . This work aims to give a unified version of these two theories. Contrary to what one could expect,

the solution is not given in terms of the limiting cases above considered (i.e.,  $u = 1$  and  $w = 1$ ), but in a rather more complicated condition, which reflects the difficulty of estimating the distribution function of the Hardy-Littlewood maximal operator with respect to general measures.

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**Referència:** Martín, J., Milman, M. Isoperimetry and symmetrization for logarithmic Sobolev inequalities. *J. Funct. Anal.*, **256** (2009), pp. 149–178.

**Abstract:** Using isoperimetry and symmetrization we provide a unified framework to study the classical and logarithmic Sobolev inequalities. In particular, we obtain new Gaussian symmetrization inequalities and connect them with logarithmic Sobolev inequalities. Our methods are very general and can be easily adapted to more general contexts.