## Publicacions més rellevants de la línia de recerca: Categories derivades

**Referència:** Costa, L. and Miró-Roig, R. M. Tilting sheaves on toric varieties. *Math. Z.* **248** (2004), 849–865.

**Abstract:** In [K], A. King states the following conjecture: Any smooth complete toric variety has a tilting bundle whose summands are line bundles. The goal of this paper is to prove King's conjecture for the following types of smooth complete toric varieties:

- (i) Any *d*-dimensional smooth complete toric variety with splitting fan.
- (ii) Any d-dimensional smooth complete toric variety with Picard number  $\leq 2$ .
- (iii) The blow up of any smooth complete minimal toric surface at T-invariants points.

**Referència:** Costa, L. and Miró-Roig, R.M. m-blocks collections and Castelnuovo-Mumford regularity. *Nagoya J. Math.*, **186** (2007), 119–155.

Abstract: The main goal of the paper is to generalize Castelnuovo-Mumford regularity for coherent sheaves on projective spaces to coherent sheaves on *n*-dimensional smooth projective varieties X with an *n*-block collection  $\mathcal{B}$  which generates the bounded derived category  $\mathcal{D}^b(\mathcal{O}_X\text{-}mod)$ . To this end, we use the theory of *n*-blocks and Beilinson type spectral sequence to define the notion of regularity of a coherent sheaf F on X with respect to the *n*-block collection  $\mathcal{B}$ . We show that the basic formal properties of the Castelnuovo-Mumford regularity of coherent sheaves over projective spaces continue to hold in this new setting and we compare our definition of regularity with previous ones. In particular, we show that in case of coherent sheaves on  $\mathbb{P}^n$  and for the *n*-block collection  $\mathcal{B} = (\mathcal{O}_{\mathbb{P}^n}, \mathcal{O}_{\mathbb{P}^n}(1), \cdots, \mathcal{O}_{\mathbb{P}^n}(n))$  on  $\mathbb{P}^n$  Castelnuovo-Mumford regularity and our new definition of regularity coincide. Finally, we carefully study the regularity of coherent sheaves on a multiprojective space  $\mathbb{P}^{n_1} \times \cdots \times \mathbb{P}^{n_r}$  with respect to a suitable  $n_1 + \cdots + n_r$ -block collection and we compare it with the multigraded variant of the Castelnuovo-Mumford regularity given by Hoffman and Wang in [HW].

Referència: Costa, L. and Miró-Roig, R.M. Geometric collections and Castelnuovo-Mumford

regularity. Proc. Cambridge Math. Soc., 143 (2007), 557-578.

Abstract: The paper begins by overviewing the basic facts on geometric exceptional collections. Then, we derive, for any coherent sheaf  $\mathcal{F}$  on a smooth projective variety with a geometric collection, two spectral sequences: the first one abuts to  $\mathcal{F}$  and the second one to its cohomology. The main goal of the paper is to generalize Castelnuovo-Mumford regularity for coherent sheaves on projective spaces to coherent sheaves on smooth projective varieties X with a geometric collection  $\sigma$ . We define the notion of regularity of a coherent sheaf  $\mathcal{F}$  on X with respect to  $\sigma$ . We show that the basic formal properties of the Castelnuovo-Mumford regularity of coherent sheaves over projective spaces continue to hold in this new setting and we show that in case of coherent sheaves on  $\mathbb{P}^n$  and for a suitable geometric collection of coherent sheaves on  $\mathbb{P}^n$  both notions of regularity coincide. Finally, we carefully study the regularity of coherent sheaves on a smooth quadric hypersurface  $Q_n \subset \mathbb{P}^{n+1}$  (n odd) with respect to a suitable geometric collection and we compare it with the Castelnuovo-Mumford regularity of their extension by zero in  $\mathbb{P}^{n+1}$ .