

**Publicacions més rellevants de la línia de recerca:  
Dinàmica combinatòria i topològica**

**Referència:** Alsedà Ll., Gautero F., Guaschi J., Los J., Mañosas F. and Mumbrú P. Patterns and minimal dynamics for graph maps. *Proceedings of the London Mathematical Society*, **91** (2) (2005), pp. 414–442.

**Abstract:** We study the rigidity problem for periodic orbits of (continuous) graph maps belonging to the same homotopy equivalence class. Since the underlying spaces are not necessarily homeomorphic, we define a new notion of pattern which enables us to compare periodic orbits of self-maps of homotopy-equivalent spaces. This definition unifies the known notions of pattern for other spaces. The two main results of the paper are as follows: given a free group endomorphism, we study the persistence under homotopy of the periodic orbits of its topological representatives, and in the irreducible case, we prove the minimality (within the homotopy class) of the set of periodic orbits of its efficient representatives.

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**Referència:** Alsedà Ll. and Ruelle S. Rotation sets for graph maps of degree 1. *Annals Institute Fourier*, **58**(4) (2008), pp. 1233–1294.

**Abstract:** For a continuous map on a topological graph containing a loop  $S$  it is possible to define the degree (with respect to the loop  $S$ ) and, for a map of degree 1, rotation numbers. We study the rotation set of these maps and the periods of periodic points having a given rotation number. We show that, if the graph has a single loop  $S$  then the set of rotation numbers of points in  $S$  has some properties similar to the rotation set of a circle map; in particular it is a compact interval and for every rational  $\alpha$  in this interval there exists a periodic point of rotation number  $\alpha$ . For a special class of maps called combed maps, the rotation set displays the same nice properties as the continuous degree one circle maps.

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**Referència:** Alsedà, Ll. and Misiurewicz, M. Attractors for unimodal quasiperiodically forced maps. *Journal of Difference Equations and Applications*, **4**(10) (2008), pp. 1175–1196.

**Abstract:** We consider unimodal quasiperiodically forced maps, that is, skew products with

irrational rotations of the circle in the base and unimodal interval maps in the fibres: the map in the fibre over  $\theta$  is a unimodal map  $f$  of the interval  $[0, 1]$  onto itself multiplied by  $g(\theta)$ , where  $g$  is a continuous function from the circle to  $[0, 1]$ . Here we consider the *pinched* case, when  $g$  attains the value 0. This case is similar to the one considered by Gerhard Keller, except that the function  $f$  in his case is increasing. Since in our case  $f$  is unimodal, the basic tools from the Keller's paper do not work in general. We prove that under some additional assumptions on the system there exists a *strange nonchaotic attractor*. It is the graph of a measurable function from the circle to  $[0, 1]$ , which is invariant, discontinuous almost everywhere and attracts almost all trajectories. Moreover, both Lyapunov exponents on this attractor are nonpositive. There are also cases when the dynamics is completely different, because one can apply the results of Jerome Buzzi implying the existence of an invariant measure absolutely continuous with respect to the Lebesgue measure (and then the attractor is some region in  $\mathbb{S}^1 \times [0, 1]$ ), and the maximal Lyapunov exponent is positive. Finally, there are cases when we can only guess what the behaviour is by performing computer experiments.

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