## Publicacions més rellevants de la línia de recerca: Optimització estructurada

**Referència:** Daniilidis, A., Sagastizábal, C., and Solodov, M., Identifying Structure of Nonsmooth Convex Function by the Bundle Techniques. *SIAM J. Optimization*, **20** (2) (2009), pp. 820–840.

Abstract: We consider the problem of minimizing nonsmooth convex functions, defined piecewise by a finite number of functions each of which is either convex quadratic or twice continuously differentiable with positive definite Hessian on the set of interest. This is a particular case of functions with primal-dual gradient structure, a notion closely related to the so-called  $\mathcal{VU}$  space decomposition: at a given point, nonsmoothness is locally restricted to the directions of the subspace  $\mathcal{V}$ , while along the subspace  $\mathcal{U}$  the behaviour of the function is twice differentiable. Constructive identification of the two subspaces is important, because it opens the way to devising fast algorithms for nonsmooth optimization (by following iteratively the manifold of smoothness, on which superlinear  $\mathcal{U}$ -Newton steps can be computed). In this work we show that for the class of functions in consideration, the information needed for this identification can be obtained from the output of a standard bundle method for computing proximal points, provided a minimizer satisfies the non-degeneracy and strong transversality conditions.

**Referència:** Bolte, J., Daniilidis, A. and Lewis, A.S., Lojasiewicz inequality for nonsmooth subanalytic functions with applications to subgradient dynamical systems. *SIAM J. Optimization*, **17** (4) (2007), pp. 1205-1223.

Abstract: Given a real-analytic function  $f : \mathbb{R}^n \to \mathbb{R}$  and a critical point  $a \in \mathbb{R}^n$ , the Lojasiewicz inequality asserts that there exists  $\theta \in [\frac{1}{2}, 1)$  such that the function  $|f - f(a)|^{\theta} ||\nabla f||^{-1}$  remains bounded around a. In this paper, we extend the above result to a wide class of nonsmooth functions (that possibly admit the value  $+\infty$ ), by establishing an analogous inequality in which the derivative  $\nabla f(x)$  can be replaced by any element  $x^*$  of the subdifferential  $\partial f(x)$  of f. Like its smooth version, this result provides new insights into the convergence aspects of subgradient-type dynamical systems. Provided that the function f is sufficiently regular (for instance, convex or lower- $C^2$ ), the bounded trajectories of the corresponding subgradient dynamical system can be shown to be of finite length. Explicit estimates of the rate of convergence are also derived.

**Referència:** Aussel, D., Daniilidis, A. and Thibault, L., Subsmooth sets: functional characterizations and related concepts *Trans. Amer. Math. Soc.*, **357** (4) (2005), pp. 1275–1301.

Abstract: Prox-regularity of a set (Poliquin-Rockafellar-Thibault, 2000), or its global version, proximal smoothness (Clarke-Stern-Wolenski, 1995) plays an important role in variational analysis, not only because it is associated with some fundamental properties as the local continuous differentiability of the function dist  $(C; \cdot)$ , or the local uniqueness of the projection mapping, but also because in the case where C is the epigraph of a locally Lipschitz function, it is equivalent to the weak convexity (lower-C<sup>2</sup> property) of the function. In this paper we provide an adapted geometrical concept, called *subsmoothness*, which permits an epigraphic characterization of the approximate convex functions (or lower-C<sup>1</sup> property). Subsmooth sets turn out to be naturally situated between the classes of prox-regular and of nearly radial sets. This latter class has been recently introduced by Lewis in 2002. We hereby relate it to the Mifflin semismooth functions.