## Publicacions més rellevants de la línia de recerca: Mètodes geomètrics en control i control òptim: anormalitat i controlabilitat

**Referència:** Barbero, M., Echeverría-Enríquez, A., Martín de Diego, D., Muñoz-Lecanda, M.C. and Román-Roy, N. "Skinner-Rusk unified formalism for optimal control systems and applications". *J. Phys. A: Math. Theor.* **40**(40) (2007) 12071-12093.

**Abstract:** A geometric approach to time-dependent optimal control problems is proposed. This formulation is based on the Skinner and Rusk formalism for Lagrangian and Hamiltonian systems. The corresponding unified formalism developed for optimal control systems allows us to formulate geometrically the necessary conditions given by a weak form of Pontryagin's Maximum Principle, provided that the differentiability with respect to controls is assumed and the space of controls is open. Furthermore, our method is also valid for implicit optimal control systems and, in particular, for the so-called descriptor systems (optimal control problems including both differential and algebraic equations).

**Referència:** Muñoz-Lecanda, M.C. and Yániz, J. "Mechanical control systems and kinematic systems". *IEEE Trans. Automat. Control* **53**(5) (2008) 1297-1302.

**Abstract:** The aim of this technical note is to analyze the equivalence between the second-order equations describing the dynamics of mechanical systems, and the associated kinematic system when dealing with nonholonomic systems with controls. If the system is fully actuated, both systems are equivalent. However, if it is underactuated an extra condition must be imposed to ensure that a weak equivalence holds. Furthermore, the notion of decoupling vector fields is generalized to a vector field distribution. This point of view may be used to obtain better solutions when a cost function is introduced to the controllability problem. The results are applied to some particular examples and to mechanical systems with symmetries.

**Referència:** Barbero, M. and Muñoz-Lecanda, M.C. "Geometric approach to Pontryagin's Maximum Principle". *Acta Appl. Math.* **108** (2009) 429-485.

Abstract: Since the second half of the 20th century, Pontryagin's Maximum Principle has been

widely discussed and used as a method to solve optimal control problems in medicine, robotics, finance, engineering, astronomy. Here, we focus on the proof and on the understanding of this Principle, using as much geometric ideas and geometric tools as possible. This approach provides a better and clearer understanding of the Principle and, in particular, of the role of the abnormal extremals. These extremals are interesting because they do not depend on the cost function, but only on the control system. Moreover, they were discarded as solutions until the nineties, when examples of strict abnormal optimal curves were found. In order to give a detailed exposition of the proof, the paper is mostly self-contained, which forces us to consider different areas in mathematics such as algebra, analysis, geometry.