Publicacions més rellevants de la línia de recerca:

Corbes i varietats abelianes modulars

Referència: González J., Lario J-C.; **Q**-curves and their Manin ideals, *American Journal of Mathematics*, **123(3)** (2001), pp. 475–503.

Abstract: Let C be an elliptic curve over $\mathbf{Q}$ and let ω denote its Néron differential (uniquely determined up to sign). According to a conjecture of Manin-Stevens, there is a modular parametrization $\pi : X_1(N) \to C$ defined over $\mathbf{Q}$ such that the so-called Manin constant $c(\pi)$ is equal to 1. The constant $c(\pi)$ is defined by the equality $\pi^*(\omega) = c(\pi) f(q) dq/q$, where f is a normalized newform of weight 2 on $\Gamma_1(N)$. In this paper we propose a generalization of the Manin constant as a certain ideal (we call it the Manin ideal) attached to modular parametrizations of elliptic curves defined over number fields. We conjecture that its value is (1), generalizing Manin-Stevens, and show the Manin ideal to be an integral ideal involving only primes dividing twice the conductor. Motivated by the Manin ideal considerations, we first study several aspects of building blocks of modular abelian varieties and modular parametrizations of **Q**-curves. Some examples are included to provide numerical evidence of the generalized conjecture, and the paper also contains the analogous items on the Manin ideals for modular building blocks of higher dimension.

Referència: Baker M., González-Jiménez E., González J., Poonen B.; Finiteness results for modular curves of genus at least 2. *American Journal of Mathematics*, **127(6)** (2005), pp. 1325–1387.

Abstract: A curve X over $\mathbf{Q}$ is modular if it is dominated by $X_1(N)$ for some N; if in addition the image of its jacobian in $J_1(N)$ is contained in the new subvariety of $J_1(N)$, then X is called a new modular curve. We prove that for each $g \geq 2$, the set of new modular curves over $\mathbf{Q}$ of genus g is finite and computable. For the computability result, we prove an algorithmic version of the de Franchis-Severi Theorem. Similar finiteness results are proved for new modular curves of bounded gonality, for new modular curves whose jacobian is a quotient of $J_0(N)^{\text{new}}$ with N divisible by a prescribed prime, and for modular curves (new or not) with levels in a restricted set. We study new modular hyperelliptic curves in detail. In particular, we find all new modular curves of genus 2 explicitly, and construct what might be the complete list of all new modular hyperelliptic curves of all genera. Finally we prove that for each field k of characteristic zero and

$g \geq 2$, the set of genus-g curves over k dominated by a Fermat curve is finite and computable.

Referència: González J., Lario J-C.; Modular elliptic directions with complex multiplication (with an application to Gross's elliptic curves $A(p)$). *Commentarii Mathematici Helvetici*, (2010), acceptat el 2009 i pendent de publicació.

Abstract: Let A_f be the abelian variety attached by Shimura to a normalized newform $f \in S_2(\Gamma_1(N))$ and assume that A_f has elliptic quotients. The paper deals with the determination of the one dimensional subspaces (elliptic directions) in $S_2(\Gamma_1(N))$ corresponding to the pullbacks of the regular differentials of all elliptic quotients of A_f. For modular elliptic curves over number fields without complex multiplication (CM), the directions were studied by the authors in [8]. The main goal of the present paper is to characterize the directions corresponding to elliptic curves with CM. Then, we apply the results obtained to the case $N = p^2$, for primes $p > 3$ and $p \equiv 3 \pmod 4$. For this case we prove that if f has CM, then all optimal elliptic quotients of A_f are also optimal in the sense that its endomorphism ring is the maximal order of $\mathbf{Q}(\sqrt{-p})$. Moreover, if f has trivial Nebentypus then all optimal quotients are Gross's elliptic curve $A(p)$ and its Galois conjugates. Among all modular parametrizations $J_0(p2) \to A(p)$, we describe a canonical one and discuss some of its properties.